Date 18/04/20 Examples on Double limits and Repeated limits.

Problem based on Double limits :→

Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$$

Solⁿ :⇒ Let $y = mx$

Here $f(x,y) = \dfrac{xy}{\sqrt{x^2+y^2}}$

put $y = mx$

$$= \frac{mx^2}{\sqrt{x^2(1+m^2)}} = \frac{mx}{\sqrt{1+m^2}}$$

$\therefore \lim_{\substack{x\to 0\\ y\to 0}} f(x,y) = \lim_{x\to 0} \dfrac{mx}{\sqrt{1+m^2}} = 0$ for all finite values of m.

2. Show that

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{2xy}{x^2+y^2}$$ does not exist.

Here $f(x,y) = \dfrac{2xy}{x^2+y^2} = \dfrac{2x\cdot mx}{x^2+m^2x^2}$ Putting $y=mx$

$$= \frac{2m}{1+m^2}$$

$$\therefore \lim_{\substack{x\to 0\\ y\to 0}} f(x,y) = \lim_{x\to 0} \frac{2mx^2}{(1+m^2)x^2} = \frac{2m}{1+m^2}$$

Which may assume different values, for m may take up any value.

it means that if we approach $(0,0)$ along a line having slope m_1, the above limit is $\dfrac{2m_1}{1+m_1^2}$

while if we approaches the point $(0,0)$ along a line having slope m_2, the above limit is $\dfrac{2m_2}{1+m_2^2}$. This the given limit does not exist.

3. Show that-

$$\lim_{\substack{x \to 0 \\ y \to 0}} \frac{xy^3}{x^2+y^6} \text{ does not Exist.}$$

Here $f(x,y) = \dfrac{xy^3}{x^2+y^6}$

If we put $x = my^3$ and let $y \to 0$

we get-

$$\lim_{\substack{x \to 0 \\ y \to 0}} \frac{xy^3}{x^2+y^6} = \lim_{y \to 0} \frac{my^6}{m^2y^6+y^6}$$

$$= \lim_{y \to 0} \frac{my^6}{(m^2+1)y^6} = \frac{m}{1+m^2}$$

Which is different for different values of m.

Hence Limit does not Exist.

4. Ex. → for the function

$$f(x,y) = y^2 \sin\frac{1}{x} \quad (x \neq 0)$$

$$f(0,0) = 0.$$

Show that $\lim_{\substack{x \to 0 \\ y \to 0}} f(x,y) = 0$

Sol - let $\epsilon > 0$ (however small) be given

Here $\left| f(x,y) - 0 \right| = \left| y^2 \sin\frac{1}{x} - 0 \right|$

$\leq y^2$ since $\sin\frac{1}{x} \leq 1$

$< \epsilon$ if $y^2 < \epsilon$ i.e. if $|y| < \sqrt{\epsilon}$

thus Given $\epsilon > 0$ however small there Exists a $\delta > 0$ such that-

$\left| y^2 \sin\frac{1}{x} \right| < \epsilon$ whatever $|y| < \delta$

$\therefore \lim_{\substack{x \to 0 \\ y \to 0}} y^2 \sin\frac{1}{x} = 0$ thus the Limit Exists at the origin.